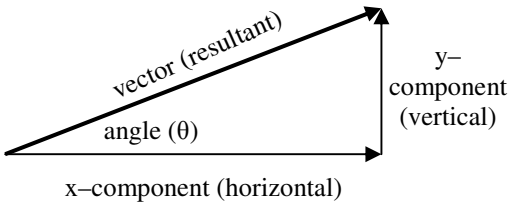
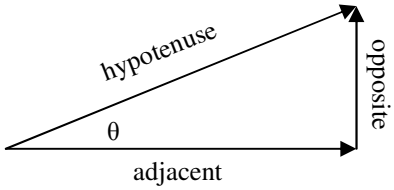
Vectors and Components

Vectors have magnitude and direction. In order to add vectors together that are not on the x or y axis, we need to break them down into their x and y components, their horizontal and vertical components. Adding is, then, simple math.



Finding vector components requires the knowledge of basic trigonometry.



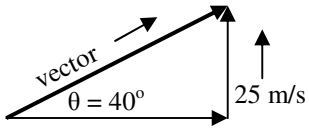
Basic trigonometry relationships:

sin θ = opposite/hypotenuse
cos θ = adjacent/hypotenuse
tan θ = opposite/ adjacent

You will be given two of these quantities; find the rest from these formulas.

Ex. 1 If a vector's direction is θ = 40° from the horizontal and the vertical component is 25 m/s, find the magnitude of the vector.

Step 1: Draw Diagram



Step 2: Assign variables

vector = hypotenuse
25 m/s = opposite
θ = 40°

Step 3: Equation

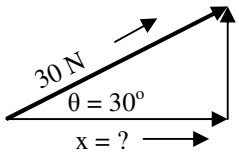
sin θ = opp./hypo.

Step 4: Solve

sin 40° = (25m/s)/hypo.
hypo. = (25 m/s)/sin 40°
hypo. = (25 m/s)/(.6428)
hypo. = 39 m/s

Ex. 2 What is the horizontal component of a 30 N force applied 30° to the horizontal.

Step 1: Draw Diagram



Step 2: Assign variables

30 N = hypotenuse
x = adjacent
θ = 30°

Step 3: Equation

cos θ = adj./hypo.

Step 4: Solve

cos 30° = x/(30 N)
(30 N)(cos 30°) = x
(30 N)(.866) = x
x = 25.98 N

Because you will find the vector components so often, it will be helpful to derive two *component equations:*

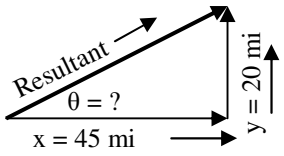
Vector Component Equations

cos θ = adjacent/hypotenuse
adjacent = hypotenuse(cosθ)

sin θ = opposite/hypotenuse
opposite = hypotenuse(sin θ)

Ex. 3 A person drives 20 miles north and 45 miles east. What is the angle of the resultant vector straight to their destination?

Step 1: Draw Diagram



Step 2: Assign variables

20 mi = opposite
45 mi = adjacent
θ = ?

Step 3: Equation

tan θ = opp./adj.

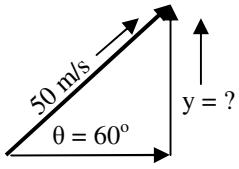
Step 4: Solve

tan θ = 20 mi/45 mi
tan θ = .4444
θ = $\tan^{-1}$(.4444)
θ = 24°

Calculator tip: for $\tan^{-1}$ push (INV)tan or (2nd)tan

Ex. 4 A plane is flying 50 m/s and 60° up into the air. How much altitude will it gain in 6 seconds?

Step 1: Draw Diagram



Step 2: Assign variables

50 m/s = hypotenuse
y = opposite
θ = 60°

Step 3: Equation

opp.= hypo(sin θ)

Step 4: Solve

y = vector(sin θ)
y = (50 m/s)(sin 60°)
y = (50 m/s)(.866)
y = 43.3 m/s

Not the final answer: this is just the y-component of the speed. We must still find distance.

Hint: "Altitude" means "height above the ground." So you need the vertical component of the speed. Then you can find distance from speed and time.

Step 5: speed equation

Vertical speed = 43.3 m/s
v = Δx/Δt

Step 6: solve

43.3m/s = Δx/(6 sec)
(6 sec)(43.3 m/sec) = Δx
259.8 m = Δx