## **Friction and Angles – Two Examples**

This page assumes you have read and understood the notes on Normal Force and Friction.

## Type 1: Object on a horizontal surface with a force applied at an angle.

*Example:* A 60 N force is applied at 40° to an 8 kg object. Will the object slide? If so, find its acceleration.

Steps:

- 1) Find the weight of the object. (Remember that  $F_w = mg$ , F is in Newtons, and that gravity always pulls straight down toward the earth.)
- 2) Resolve all vectors into their x and y components. Here  $F_x = F(\cos\theta)$ ;  $F_y = F(\sin\theta)$  (if a force is already along an axis, skip this step and put it in the x or y axis).
- 3) Find the normal force. Remember that  $\Sigma F_{up} = \Sigma F_{down}$ . (If there is a force pulling up in addition to  $F_n$ ,  $F_n$  will be less than Fg.)
- 4) Find  $F_s$  and  $F_k$ . (Remember that  $\mu_s = F_s/F_n$  and  $\mu_k = F_k/F_n$ .)
- 5) Is the object moving? (Is  $F_x > F_s$ ?) (If no, you can stop here.)
- 6) Find "a" from  $\Sigma F$  = ma. (But because it is moving, you must use  $F_k$ , not  $F_{s}$ .) So  $F_x F_k$  = ma.

Big Hints for Any Friction Problem with Angles1) Draw accurate and neat diagrams.2) Keep x and y directions separate.



- 1)  $F_g = mg = 8kg(-10m/s^2) = -80 N (down)$
- 2)  $F_x = F(\cos\theta) = 60N(\cos 40^\circ) = 46 \text{ N}$  $F_y = F(\sin\theta) = 60N(\sin 40^\circ) = 38.6 \text{ N}$
- 3)  $\Sigma F_{up} = \Sigma F_{down}$   $F_n + F_y = F_g$   $F_n = F_g - F_y = 80N - 38.6N$  $F_n = 41.4N$
- $\begin{array}{ll} 4) & F_s = \mu_s F_n & F_k = \mu_k F_n \\ F_s = .5(41.4N) & F_k = .25(41.4N) \\ F_s = 20.7N & F_k = 10.4N \end{array}$
- 5) Is moving, because  $F_x > F_s$  (46N > 20.7N)
- 6)  $\Sigma F = ma$  (but use  $F_k$ , not  $F_s$ )  $F_x - F_k = ma$ 46N - 10.4N = (8 kg)a

## Type 2: Object on a tilted ramp.

Example: A 5 kg object is on a 30° ramp.  $\mu_s = .6$  $\mu_k = .35$  Will it slide? If so, find its acceleration.

Steps:

- 1) Find the weight of the object. (Remember that  $F_w = mg$ , F is in Newtons, and that gravity always pulls straight down toward the earth.)
- $\begin{array}{ll} \mbox{2)} & \mbox{Resolve } F_g \mbox{ into its components parallel and} \\ & \mbox{perpendicular to the ramp.} \\ & \mbox{Here } F_{gx} = Fg(sin\theta); \mbox{ } F_{gy} = F_g(cos\theta) \end{array}$
- 3) Find the normal force. Remember that  $\Sigma F_{up} = \Sigma F_{down}$  in the y-direction (perpendicular to the ramp) here Fn = Fgy.
- 4) Find  $F_s$  and  $F_k$ .
- 5) Is the object moving? (Is  $F_{gx} > F_s$ ?) (If no, you can stop here.)
- 6) Find "a" from ΣF = ma. (But because it is moving, you must use Fk, not Fs.) So F<sub>x</sub> - F<sub>k</sub> = ma.



- 1)  $F_g = mg = 5kg(-10m/s^2) = -50 N$  (neg. means down)
- 2)  $F_x = F(\sin\theta) = 50N(\sin 30^\circ) = 25 N$  $F_y = F(\cos\theta) = 50N(\cos 30^\circ) = 43.3 N$
- 3)  $\Sigma F_{up} = \Sigma F_{down}$ 
  - $F_n = F_{gy} \qquad F_n = 43.3N$
- $\begin{array}{ll} \text{4)} & F_{s} = \mu_{s}F_{n} & F_{k} = \mu_{k}F_{n} \\ F_{s} = .45(43.3\text{N}) & F_{k} = .2(43.3\text{N}) \\ F_{s} = 19.5\text{N} & F_{k} = 8.7\text{N} \end{array}$
- 5) It is moving, because  $F_x > F_s$  (25N > 19.5N)
- 6)  $\Sigma F = ma$  (but use  $F_k$ , not  $F_s$ )  $F_x - F_k = ma$  25N - 8.7N = (5 kg)a $a = 16.3N/5\text{kg} = 3.26 \text{ m/s}^2$