## Friction and Angles - Two Examples

This page assumes you have read and understood the notes on Normal Force and Friction.

## Type 1: Object on a horizontal surface with a force applied at an angle.

Example: A 60 N force is applied at $40^{\circ}$ to an 8 kg object. Will the object slide? If so, find its acceleration.

Steps:

1) Find the weight of the object. (Remember that $\mathrm{F}_{\mathrm{w}}=\mathrm{mg}$, $F$ is in Newtons, and that gravity always pulls straight down toward the earth.)
2) Resolve all vectors into their $x$ and $y$ components. Here $\mathrm{F}_{\mathrm{x}}=\mathrm{F}(\cos \theta) ; \mathrm{F}_{\mathrm{y}}=\mathrm{F}(\sin \theta)$ (if a force is already along an axis, skip this step and put it in the x or y axis).
3) Find the normal force. Remember that $\Sigma \mathrm{F}_{\text {up }}=\Sigma \mathrm{F}_{\text {down }}$. (If there is a force pulling up in addition to $\mathrm{F}_{\mathrm{n}}, \mathrm{F}_{\mathrm{n}}$ will be less than Fg.)
4) Find $F_{s}$ and $F_{k}$.
(Remember that $\mu_{\mathrm{s}}=\mathrm{F}_{\mathrm{s}} / \mathrm{F}_{\mathrm{n}}$ and $\mu_{\mathrm{k}}=\mathrm{F}_{\mathrm{k}} / \mathrm{F}_{\mathrm{n}}$.)
5) Is the object moving? (Is $\mathrm{F}_{\mathrm{x}}>\mathrm{F}_{\mathrm{s}}$ ?) (If no, you can stop here.)
6) Find "a" from $\Sigma \mathrm{F}=$ ma. (But because it is moving, you must use $F_{k}$, not $F_{s}$.) So $F_{x}-F_{k}=m$.

Big Hints for Any Friction Problem with Angles

1) Draw accurate and neat diagrams.
2) Keep $x$ and $y$ directions separate.

3) $\mathrm{F}_{\mathrm{g}}=\mathrm{mg}=8 \mathrm{~kg}\left(-10 \mathrm{~m} / \mathrm{s}^{2}\right)=-80 \mathrm{~N}$ (down)
4) $\mathrm{F}_{\mathrm{x}}=\mathrm{F}(\cos \theta)=60 \mathrm{~N}\left(\cos 40^{\circ}\right)=46 \mathrm{~N}$ $\mathrm{F}_{\mathrm{y}}=\mathrm{F}(\sin \theta)=60 \mathrm{~N}\left(\sin 40^{\circ}\right)=38.6 \mathrm{~N}$
5) $\Sigma \mathrm{F}_{\text {up }}=\Sigma \mathrm{F}_{\text {down }}$
$\mathrm{F}_{\mathrm{n}}+\mathrm{F}_{\mathrm{y}}=\mathrm{F}_{\mathrm{g}} \quad \mathrm{F}_{\mathrm{n}}=\mathrm{F}_{\mathrm{g}}-\mathrm{F}_{\mathrm{y}}=80 \mathrm{~N}-38.6 \mathrm{~N}$
$\mathrm{F}_{\mathrm{n}}=41.4 \mathrm{~N}$
6) $\mathrm{F}_{\mathrm{s}}=\mu_{\mathrm{s}} \mathrm{F}_{\mathrm{n}} \quad \mathrm{F}_{\mathrm{k}}=\mu_{\mathrm{k}} \mathrm{F}_{\mathrm{n}}$
$\mathrm{F}_{\mathrm{s}}=.5(41.4 \mathrm{~N}) \quad \mathrm{F}_{\mathrm{k}}=.25(41.4 \mathrm{~N})$
$\mathrm{F}_{\mathrm{s}}=20.7 \mathrm{~N}$
$\mathrm{F}_{\mathrm{k}}=10.4 \mathrm{~N}$
7) Is moving, because $F_{x}>F_{s}(46 N>20.7 N)$
8) $\quad \Sigma \mathrm{F}=$ ma (but use $\mathrm{F}_{\mathrm{k}}, \operatorname{not} \mathrm{F}_{\mathrm{s}}$ )
$\mathrm{F}_{\mathrm{x}}-\mathrm{F}_{\mathrm{k}}=\mathrm{ma}$
$46 \mathrm{~N}-10.4 \mathrm{~N}=(8 \mathrm{~kg}) \mathrm{a}$

## Type 2: Object on a tilted ramp.

Example: A 5 kg object is on a $30^{\circ} \mathrm{ramp} . \mu_{s}=.6$
$\mu_{k}=.35$ Will it slide? If so, find its acceleration.
Steps:

1) Find the weight of the object. (Remember that $\mathrm{F}_{\mathrm{w}}=\mathrm{mg}, \mathrm{F}$ is in Newtons, and that gravity always pulls straight down toward the earth.)
2) Resolve $\mathrm{F}_{\mathrm{g}}$ into its components parallel and perpendicular to the ramp.
Here $\mathrm{F}_{\mathrm{gx}}=\mathrm{Fg}(\sin \theta) ; \mathrm{F}_{\mathrm{gy}}=\mathrm{F}_{\mathrm{g}}(\cos \theta)$
3) Find the normal force. Remember that $\Sigma \mathrm{F}_{\text {up }}=\Sigma \mathrm{F}_{\text {down }}$ in the y-direction (perpendicular to the ramp) here $\mathrm{Fn}=\mathrm{Fgy}$.
4) Find $F_{s}$ and $F_{k}$.
5) Is the object moving? (Is $\mathrm{F}_{\mathrm{gx}}>\mathrm{F}_{\mathrm{s}}$ ?) (If no, you can stop here.)
6) Find " a " from $\Sigma \mathrm{F}=\mathrm{ma}$. (But because it is moving, you must use Fk, not Fs.)
So $\mathrm{F}_{\mathrm{x}}-\mathrm{F}_{\mathrm{k}}=\mathrm{ma}$.

7) $\mathrm{F}_{\mathrm{g}}=\mathrm{mg}=5 \mathrm{~kg}\left(-10 \mathrm{~m} / \mathrm{s}^{2}\right)=-50 \mathrm{~N}$ (neg. means down)
8) $\mathrm{F}_{\mathrm{x}}=\mathrm{F}(\sin \theta)=50 \mathrm{~N}\left(\sin 30^{\circ}\right)=25 \mathrm{~N}$
$\mathrm{F}_{\mathrm{y}}=\mathrm{F}(\cos \theta)=50 \mathrm{~N}\left(\cos 30^{\circ}\right)=43.3 \mathrm{~N}$
9) $\Sigma \mathrm{F}_{\mathrm{up}}=\Sigma \mathrm{F}_{\text {down }}$ $\mathrm{F}_{\mathrm{n}}=\mathrm{F}_{\mathrm{gy}} \quad \mathrm{F}_{\mathrm{n}}=43.3 \mathrm{~N}$
10) $\mathrm{F}_{\mathrm{s}}=\mu_{\mathrm{s}} \mathrm{F}_{\mathrm{n}} \quad \mathrm{F}_{\mathrm{k}}=\mu_{\mathrm{k}} \mathrm{F}_{\mathrm{n}}$ $\mathrm{F}_{\mathrm{s}}=.45(43.3 \mathrm{~N}) \quad \mathrm{F}_{\mathrm{k}}=.2(43.3 \mathrm{~N})$ $\mathrm{F}_{\mathrm{s}}=19.5 \mathrm{~N} \quad \mathrm{~F}_{\mathrm{k}}=8.7 \mathrm{~N}$
11) It is moving, because $F_{x}>F_{s}(25 N>19.5 N)$
12) $\quad \Sigma \mathrm{F}=$ ma (but use $\mathrm{F}_{\mathrm{k}}$, not $\mathrm{F}_{\mathrm{s}}$ )
$\mathrm{F}_{\mathrm{x}}-\mathrm{F}_{\mathrm{k}}=\mathrm{ma}$
$25 \mathrm{~N}-8.7 \mathrm{~N}=(5 \mathrm{~kg}) \mathrm{a}$ $\mathrm{a}=16.3 \mathrm{~N} / 5 \mathrm{~kg}=3.26 \mathrm{~m} / \mathrm{s}^{2}$
