This print-out should have 19 questions. Multiple-choice questions may continue on the next column or page – find all choices before answering. The due time is Central time.

Plan a Trip

02:03, trigonometry, numeric, > 1 min, normal.

001

You plan a trip on which you want to average a speed of 90 km/h. You cover the first half of the distance at an average speed of only 48 km/h.

What must be your average speed in the second half of the trip to meet your goal?

Correct answer: 720 km/h.

Explanation:

Basic Concepts

If d is the total distance covered, then each half of the trip will cover a distance of $\frac{d}{2}$.

For the first half of the trip,

$$t_1 = \frac{d_1}{v_1} = \frac{\frac{1}{2}\,d}{v_1} = \frac{d}{2\,v_1}$$

For the second half of the trip,

$$t_2 = \frac{d_2}{v_2} = \frac{\frac{1}{2}\,d}{v_2} = \frac{d}{2\,v_2}$$

For the entire trip,

$$t = t_1 + t_2$$

$$\frac{d}{v} = \frac{d}{2\,v_1} + \frac{d}{2\,v_2}$$

Multiplying by the LCD of $(2\,v\,v_1\,v_2)$ yields

$$2v_1\,v_2\,d = v\,v_2\,d + v\,v_1\,d$$

$$2\,v_1\,v_2 = v\,v_2 + v\,v_1$$

$$2\,v_1\,v_2 - v\,v_2 = v\,v_1$$

Solving for v_2 we have

$$\begin{aligned} v_2 &= \frac{v\,v_1}{2\,v_1 - v} \\ &= \frac{(90\text{ km/h})(48\text{ km/h})}{2(48\text{ km/h}) - (90\text{ km/h})} \\ &= 720\text{ km/h}\,. \end{aligned}$$

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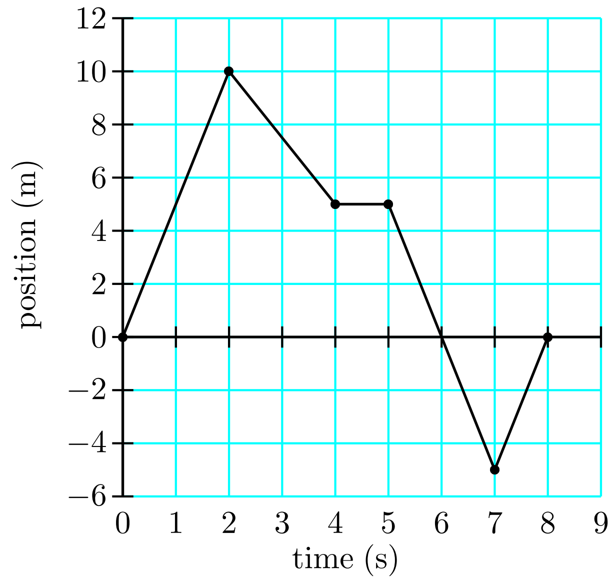
keywords:

Serway CP 02 15

02:04, trigonometry, numeric, > 1 min, normal.

002

The position versus time for a certain object moving along the x-axis is shown. The object's initial position is 0 m.



Find the instantaneous velocity at 1 s.

Correct answer: 5 m/s.

Explanation:

The instantaneous velocity is the slope of the tangent line at that point.

$$v = \frac{(10\text{ m}) - (0\text{ m})}{(2\text{ s}) - (0\text{ s})} = \boxed{5\text{ m/s}}\,.$$

003

Find the instantaneous velocity at 3 s.

Correct answer: −2.5 m/s.

Explanation:

$$v = \frac{(5\text{ m}) - (10\text{ m})}{(4\text{ s}) - (2\text{ s})} = \boxed{-2.5\text{ m/s}}\,.$$

004

Find the instantaneous velocity at 4.5 s.

Correct answer: 0 m/s.

Explanation:

$$v = \frac{(5\text{ m}) - (5\text{ m})}{(5\text{ s}) - (4\text{ s})} = \boxed{0\text{ m/s}}.$$

005

Find the instantaneous velocity at 7.5 s.
Correct answer: 5 m/s.
Explanation:

$$v = \frac{(0\text{ m}) - (-5\text{ m})}{(8\text{ s}) - (7\text{ s})} = \boxed{5\text{ m/s}}.$$

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keywords:

Acceleration

02:05, trigonometry, numeric, > 1 min, normal.

006

A car travels in a straight line for 2.5 h at a constant speed of 51 km/h .

What is its acceleration?
Correct answer: 0 m/s^2.
Explanation:

The acceleration is zero since there is no change in velocity.

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keywords:

Accelerating Car

02:06, trigonometry, numeric, > 1 min, normal.

007

A car starts from rest and accelerates for 2 s with an acceleration of 1 m/s^2.

How far does it travel?
Correct answer: 2 m.
Explanation:

The motion of the car is uniformly accelerated motion in a straight line. We can use

$$x = x_0 + v_0\, t + \frac{1}{2} a\, t^2 = \frac{1}{2} a\, t^2$$

since $x_0 = 0$ and $v_0 = 0$. Thus

$$\begin{aligned} d &= \frac{1}{2}(1\text{ m/s}^2)(2\text{ s})^2 \\ &= 2\text{ m} \end{aligned}$$

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keywords:

Bullet Acceleration

02:06, trigonometry, numeric, > 1 min, normal.

008

The barrel of a rifle has a length of 0.9 m. A bullet leaves the muzzle of a rifle with a speed of 600 m/s.

Note: A bullet in a rifle barrel does not have constant acceleration, however, constant acceleration is assumed for this problem.

What is the acceleration of the bullet while in the barrel?
Correct answer: 200000 m/s^2.
Explanation:

The initial velocity $v_i = 0$. Under the assumption that the acceleration of the bullet was constant while in the barrel,

$$v_f^2 = v_i^2 + 2\, a\, d = 2\, a\, \ell\, .$$

Thus

$$\begin{aligned} a &= \frac{v_f^2}{2\,\ell} \\ &= \frac{(600\text{ m/s})^2}{2\,(0.9\text{ m})} \\ &= 200000\text{ m/s}^2\, . \end{aligned}$$

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keywords:

Car Acceleration

02:06, trigonometry, numeric, > 1 min, normal.

009

A car increases its velocity from zero to 60 km/h in 8 s.

What is its acceleration?
Correct answer: 2.08333 m/s^2.
Explanation:

$$v_f = v_o + at$$

The initial velocity $v_o = 0$, so

$$v_f = v_o + at = at$$

$$a = \frac{v}{t}$$

Dimensional analysis for a:

$$\frac{\text{km}}{\text{h}} \div \frac{\text{s}}{1} = \frac{\text{km}}{\text{h}} \cdot \frac{1}{\text{s}} \cdot \frac{1\,\text{h}}{3600\,\text{s}} \cdot \frac{1000\,\text{m}}{1\,\text{km}} = \text{m/s}^2$$

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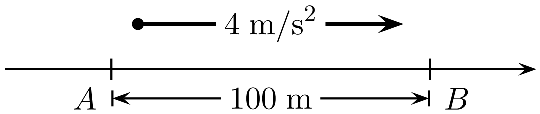
keywords:

Car and Checkpoints 01

02:06, calculus, numeric, > 1 min, normal.

010

Consider a car which is traveling along a straight road with constant acceleration a. There are two checkpoints A and B which are a distance 100 m apart. The time it takes for the car to travel from A to B is 5 s.



Find the velocity difference, $\Delta v = v_B - v_A$ between the two checkpoints.

1. $\Delta v = a\,\Delta t$ **correct**

2. $\Delta v = 2\,a\,\Delta t$

3. $\Delta v = \frac{1}{2}\,a\,\Delta t$

4. $\Delta v = \frac{1}{a\,\Delta t}$

5. $\Delta v = \frac{1}{2\,a\,\Delta t}$

6. $\Delta v = \frac{2}{a\,\Delta t}$

7. $\Delta v = -a\,\Delta t$

8. $\Delta v = -2\,a\,\Delta t$

9. $\Delta v = -\frac{1}{2}\,a\,\Delta t$

Explanation:

Basic Concepts: Constant acceleration:

$$v = v_0 + a\,t\,.$$

The average velocity $\bar{v}$ is

$$\bar{v} = \frac{\Delta x}{\Delta t} = \frac{v_0 + v_1}{2},$$

where the last equality is valid only when the acceleration is constant.

$$\begin{aligned} \text{Let}: \quad \Delta t &= 5\text{ s}\,, \\ a &= 4\text{ m/s}^2\,, \quad \text{and} \\ d &= 100\text{ m}\,. \end{aligned}$$

Solution: By definition, at any time, a is the slope of the velocity curve at that time. When a is constant, the velocity curve has a constant slope, so

$$a = \frac{dv}{dt} = \frac{\Delta v}{\Delta t}\,,$$

or

$$\Delta v = a\Delta t\,,$$

where Δt covers a finite time interval. For the explanation below, we will also write it as

$$\begin{aligned} \Delta v &= v_B - v_A \\ &= a\,\Delta t \qquad (1) \\ &= (4\text{ m/s}^2)\,(5\text{ s}) \\ &= 20\text{ m/s}\,. \end{aligned}$$

011

Find the velocity v_B for the case where the acceleration is 4 m/s^2.

Correct answer: 30 m/s.

Explanation:

Let Δx be the distance between AB. The average velocity $\bar{v}$ in the interval from A to B is

$$\bar{v} = \frac{\Delta x}{\Delta t} = \frac{(100\text{ m})}{(5\text{ s})} = 20\text{ m/s}\,.$$

The average velocity for constant acceleration is also given by

$$\bar{v} = \frac{v_A + v_B}{2}\,. \qquad (2)$$

Armed with equations (1) and (2), we can solve for v_B to find

$$\begin{aligned} v_B &= \bar{v} + \frac{\Delta v}{2} \\ &= (20\text{ m/s}) + \frac{(20\text{ m/s})}{2} \\ &= 30\text{ m/s}\,. \end{aligned}$$

keywords:

Holt SF 02C 02

02:06, highSchool, numeric, > 1 min, wording-variable.

012

When Maggie applies the brakes of her car, the car slows uniformly from 15.0 m/s to 0 m/s in 2.50 s.

How many meters before a stop sign must she apply her brakes in order to stop at the sign?

Correct answer: 18.75 m.

Explanation:

Basic Concept:

$$\Delta x = \frac{1}{2}\left(v_i + v_f\right)\Delta t$$

Given:

$$\begin{aligned} v_i &= 15.0 \text{ m/s} \\ v_f &= 0 \text{ m/s} \\ \Delta t &= 2.50 \text{ s} \end{aligned}$$

Solution: Because $v_f = 0$ m/s, the equation simplifies to

$$\begin{aligned} \Delta x &= \frac{1}{2} v_i \Delta t \\ &= \frac{1}{2}\left(15 \text{ m/s}\right)\left(2.5 \text{ s}\right) \\ &= 18.75 \text{ m} \end{aligned}$$

keywords:

Holt SF 02C 04

02:06, highSchool, numeric, > 1 min, normal.

013

A driver in a car traveling at a speed of 78 km/h sees a cat 101 m away on the road.

How long will it take for the car to accelerate uniformly to a stop in exactly 99 m?

Correct answer: 9.13846 s.

Explanation:

Basic Concepts:

$$\Delta x = \frac{1}{2}\left(v_i + v_f\right)\Delta t$$

$$1\,\text{km} = 1000\,\text{m}$$

$$1\,\text{h} = 3600\,\text{s}$$

Given:

$$\begin{aligned} v_i &= 78 \text{ km/h} \\ v_f &= 0 \text{ km/h} \\ \Delta x &= 99 \text{ m} \end{aligned}$$

Solution: Because $v_f = 0$ km/h, the equation simplifies to

$$\begin{aligned} \Delta x &= \frac{1}{2} v_i \Delta t \\ \Delta t &= \frac{2\Delta x}{v_i} \\ &= \frac{2(99 \text{ m})}{78 \text{ km/h}} \cdot \frac{1\,\text{km}}{1000\,\text{m}} \cdot \frac{3600\,\text{s}}{1\,\text{h}} \\ &= 9.13846 \text{ s} \end{aligned}$$

keywords:

Holt SF 02Rev 24

02:06, highSchool, numeric, > 1 min, wording-variable.

014

A ball initially at rest rolls down a hill with an acceleration of 3.3 m/s^2.

If it accelerates for 7.5 s, how far will it move?

Correct answer: 92.8125 m.

Explanation:

Basic Concept:

$$\Delta x = v_i \Delta t + \frac{1}{2} a (\Delta t)^2$$

Given:

$$\begin{aligned} v_i &= 0\,\text{m/s} \\ a &= 3.3\,\text{m/s}^2 \\ \Delta t &= 7.5\,\text{s} \end{aligned}$$

Solution: Because $v_i = 0$ m/s, the equation simplifies to

$$\begin{aligned} \Delta x &= \frac{1}{2}(\Delta t)^2 \\ &= \frac{1}{2}\left(3.3 \text{ m/s}^2\right)\left(7.5 \text{ s}\right)^2 \\ &= 92.8125 \text{ m} \end{aligned}$$

keywords:

Return to the Start

02:06, trigonometry, numeric, > 1 min, normal.

015

A particle, traveling at 6 m/s decelerates at 1.1 m/s^2.

How long will it take to get back to the starting point?

Correct answer: 10.9091 s.

Explanation:

When the particle has returned to the starting point, $s_f = s_0$. A deceleration is a negative acceleration, so

$$\begin{aligned} s_f &= s_0 + v_0 t - \frac{1}{2} a t^2 \\ 0 &= v_0 t - \frac{1}{2} a t^2 \\ 0 &= 2 v_0 t - a t^2 \\ 0 &= t(2 v_0 - a t) \\ t &= \frac{2 v_0}{a} \end{aligned}$$

The trivial solution $t = 0$ must be rejected.

keywords:

Car Passing a Train

02:06, trigonometry, numeric, > 1 min, normal.

016

A train is moving parallel and adjacent to a highway with a constant speed of 33 m/s. Initially a car is 50 m behind the train, traveling in the same direction as the train at 48 m/s, and accelerating at 2 m/s^2.

What is the speed of the car just as it passes the train?

Correct answer: 53.6155 m/s.

Explanation:

Basic Concepts: Solution: Let (using *lower case to be initial* and *upper case to be final*)

$$\begin{aligned} t_t &= 0 \text{ s} = \text{initial time} \\ t &= t_T = \text{final time} \\ \ell &= \ell_t = 50 \text{ m} \\ \ell_T &= 0 \text{ m} = \text{final separation} \\ v_t &= v_T = 33 \text{ m/s} \\ v_c &= 48 \text{ m/s} \\ v_C &= \text{final speed of car} \\ v_r &= \text{initial relative speed of car to train} \\ v_R &= \text{final relative speed of car to train} \\ a &= 2 \text{ m/s}^2 \end{aligned}$$

We need to calculate the velocity of the car when it passes the train. In the frame of reference of the train, the initial speed of the car v_r relative to the train is

$$\begin{aligned} v_r &= v_c - v_t \\ &= 48 \text{ m/s} - 33 \text{ m/s} = 15 \text{ m/s} \end{aligned} \quad (1)$$

When the car reaches the train, the speed of the car v_R relative to the train is

$$v_R = v_C - v_t \quad (2)$$

The velocity v_R can be obtained from the kinematic relation

$$v_R^2 - v_r^2 = 2\, a\, \ell \quad (3)$$

Where $a = 2 \text{ m/s}^2$, $\ell = 50$ m , and $v_r = 15$ m/s from eqn (1). Thus,

$$\begin{aligned} v_R &= \sqrt{v_r^2 + 2\, a\, \ell} \\ &= \sqrt{(15 \text{ m/s})^2 + 2(2 \text{ m/s}^2)(50 \text{ m})} \\ &= 20.6155 \text{ m/s} \end{aligned} \quad (4)$$

The velocity of the car with respect to the ground just after passing the train is then, from the relation obtained in eqn (2),

$$\begin{aligned} v_C &= v_R + v_t \\ &= 20.6155 \text{ m/s} + 33 \text{ m/s} \\ &= 53.6155 \text{ m/s} \end{aligned} \quad (5)$$

Working the problem this way gives you experience in making coordinate transformations (changing into the coordinate system of the moving train). This makes the problem easy since it consists of a few simple steps. Many ideas in physics are more easily explained in a different coordinate system from that in which we may be used to.

Alternative explanation: To determine the velocity of the car at the moment when it just passes the train: For simplicity, we can always set $t_0 = 0$ and set the origin of the coordinate system to be at the position of the car. Then the train is at $x_t = \ell = 50$ m. At a later time t, the position of the car will be given by:

$$x_C = v_c\, t + \frac{a}{2}\, t^2 \tag{6}$$

while the position of the train is given by:

$$x_T = \ell + v_t\, t \tag{7}$$

When the car catches up with the train at t, one must have $x_C = x_T$, or,

$$v_c\, t + \frac{a}{2}\, t^2 = \ell + v_t\, t \tag{8}$$

$$\left[\frac{a}{2}\right]\, t^2 - [v_t - v_c]\, t - \ell = 0 \tag{9}$$

Solve this quadratic equation for the time t

$$\begin{aligned} t &= \frac{v_t - v_c + \sqrt{(v_t - v_c)^2 + 4\,\frac{a}{2}\,\ell}}{a} \qquad (10) \\ &= 2.80776 \text{ s}. \end{aligned}$$

At this moment t, the velocity of the car v_C is then given by:

$$\begin{aligned} v_C &= v_c + a\, t \qquad (11) \\ v_C &= (48 \text{ m/s}) + (2 \text{ m/s}^2)(2.80776 \text{ s}) \\ &= 53.6155 \text{ m/s} \end{aligned}$$

where v_c is the velocity of the car at $t = 0$.

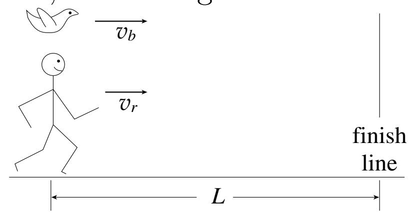
keywords:

Bird and Runner

02:02, trigonometry, numeric, > 1 min, normal.

017

A runner is jogging at a steady $v_r = 5$ km/hr. When the runner is $L = 5$ km from the finish line, a bird begins flying from the runner to the finish line at $v_b = 10$ km/hr (2 times as fast as the runner). When the bird reaches the finish line, it turns around and flies back to the runner. Even though the bird is a dodo, we will assume that it occupies only one point in space, *i.e.*, a zero length bird.



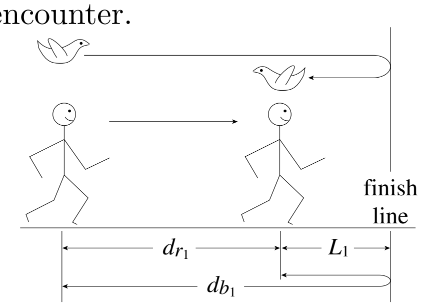
How far does the bird travel?

Correct answer: 6.66667 km.

Explanation:

Let, dodo birds fly, and

d_r be the distance the runner travels.
d_b be the distance the bird travels.
v_r be the speed of the runner.
v_b be the speed of the bird.
$L = d_r$ be the original distance to the finish line.
L_1 be the distance to the finish line after the first encounter.
$\vdots$
L_i be the distance to the finish line after the i^{th} encounter.



Since the bird travels 2 times as fast as the runner at the first meeting between the bird and runner,

$$d_{b_1} = 2\, d_{r_1}\,. \tag{1}$$

The sum of the bird's and runner's distances is 2 times L.

$$d_{b_1} + d_{r_1} = 2\, L\,. \tag{2}$$

Therefore, substituting for d_{b_1} from Eq. (1)

$$d_{r_1} + 2\, d_{r_1} = 2\, L$$

$$d_{r_1} = \frac{2}{3}\, L = \frac{2}{3}\,(5 \text{ km}) = 3.33333 \text{ km}\,. \tag{3}$$

Thus the distance the bird flies is

$$\begin{aligned} d_{b_1} &= 2\, d_{r_1} = \frac{4}{3}\, L \\ &= \frac{4}{3}\,(5 \text{ km}) = 6.66667 \text{ km}\,, \qquad (4) \end{aligned}$$

and the distance for the runner to travel after this first encounter is

$$L_1 = \frac{1}{3}\,L = \frac{1}{3}\,(5\text{ km}) = 1.66667\text{ km}\,.$$

018

After this first encounter, the bird then turns around and flies from the runner back to the finish line, turns around again and flies back to the runner. The bird repeats the back and forth trips until the runner reaches the finish line.

How far does the bird travel from the beginning? (*i.e.*, include the distance traveled to the first encounter)

Correct answer: 10 km.

Explanation:

Repeating this scenario a second time the distance for the runner to travel after the second encounter is

$$L_2 = \frac{1}{3}\,L_1 = \left(\frac{1}{3}\right)^2 L\,,$$

and the third time

$$L_3 = \frac{1}{3}\,L_2 = \left(\frac{1}{3}\right)^3 L\,,$$

and the i^{th} time

$$L_i = \frac{1}{3}\,L_{i-1} = \left(\frac{1}{3}\right)^i L\,. \qquad (5)$$

Note: The distance the bird travels between the $(i-1)^{th}$ and i^{th} time is [see Eq. (4)]

$$d_{b_i} = \frac{4}{3}\,L\left(\frac{1}{3}\right)^i \qquad (6)$$

and summing over all terms d_{b_i}

$$d_b = \sum_{i=0}^{\infty} d_{b_i} = \frac{4}{3}\,L\left[\sum_{i=0}^{\infty}\left(\frac{1}{3}\right)^i\right] \qquad (7)$$

$$= \frac{4}{3}\,L\left[1 + \frac{1}{3} + \left(\frac{1}{3}\right)^2 + \left(\frac{1}{3}\right)^3 + \left(\frac{1}{3}\right)^4 + \left(\frac{1}{3}\right)^5 + \left(\frac{1}{3}\right)^6 + \cdots\right] \qquad (8)$$

Or, by factoring $\frac{1}{3}$ from the second term on

$$d_b = \frac{4}{3}\,L\left\{1 + \frac{1}{3}\left[1 + \frac{1}{3} + \left(\frac{1}{3}\right)^2 + \left(\frac{1}{3}\right)^3 + \left(\frac{1}{3}\right)^4 + \left(\frac{1}{3}\right)^5 + \cdots\right]\right\} \qquad (9)$$

By comparing Eq. (8) with (9), and generalizing ($\ell = 1$, and $k = 3$), the infinite series

$$\sum_{i=0}^{\infty}\left(\frac{\ell}{k}\right)^i = 1 + \frac{\ell}{k}\sum_{i=0}^{\infty}\left(\frac{\ell}{k}\right)^i \qquad (10)$$

then solving Eq. (10) for $\displaystyle\sum_{i=0}^{\infty}\left(\frac{\ell}{k}\right)^i$

$$\sum_{i=0}^{\infty}\left(\frac{\ell}{k}\right)^i = \left.\frac{k}{k-\ell}\right|_{k=3,\,\ell=1} \qquad (11)$$

$$\sum_{i=0}^{\infty}\left(\frac{1}{3}\right)^i = \frac{3}{3-1} = \frac{3}{2}\,.$$

Therefore [from Eq. (7)]

$$\begin{aligned} d_b &= \frac{4}{3}\,L\sum_{i=0}^{\infty}\left(\frac{1}{3}\right)^i \\ &= \frac{4}{3}\,L\,\frac{3}{2} = 2\,L \\ &= 2\,(5\text{ km}) = 10\text{ km}\,. \end{aligned}$$

Elegant Alternative Solution: The bird will travel 2 times as far as the runner in the same time. Since the bird and jogger travel for the same length of time, the bird will travel

$$d_b = 2 \times L = 2\,(5\text{ km}) = 10\text{ km}$$

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keywords:

Overtaking a Particle

02:06, trigonometry, numeric, > 1 min, normal.

019

Two particles are at the same point at the same time, moving in the same direction. Particle A has an initial velocity of 8.7 m/s and an acceleration of 3 m/s^2. Particle B has an initial velocity of 2.3 m/s and an acceleration of 4.7 m/s^2.

At what time will B pass A?

Correct answer: 7.52941 s.

Explanation:

Basic Concept:

$$s = s_0 + v_0 t + \frac{1}{2} a t^2$$

Solution:

For particle A, $s_{0A} = 0$, so

$$s_A = v_{0A} t + \frac{1}{2} a_A t^2$$

For particle B, $s_{0B} = 0$, so

$$s_B = v_{0B} t + \frac{1}{2} a_B t^2$$

When B overtakes A, the positions are the same, so

$$v_{0A} t + \frac{1}{2} a_A t^2 = v_{0B} t + \frac{1}{2} a_B t^2$$

$$2 v_{0A} t + a_A t^2 = 2 v_{0B} t + a_B t^2$$

$$a_A t^2 - a_B t^2 + 2 v_{0A} t - 2 v_{0B} t = 0$$

$$t \left(a_A t - a_B t + 2 v_{0A} - 2 v_{0B} \right) = 0$$

The solutions are

$$t = 0$$

which tells us that they were at the same position when t=0 and

$$(a_A - a_B) t + 2 \left(v_{0A} - v_{0B} \right) = 0$$

$$\Longrightarrow t = \frac{2 \left(v_{0B} - v_{0A} \right)}{a_A - a_B}$$

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keywords: