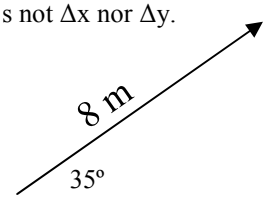
Kinematics at an Angle

You should already know: Δx = horizontal displacement (change of position in the x direction).
Δy = vertical displacement (change of position in the y direction).

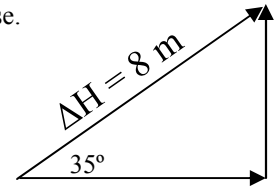
But not all objects are moving vertically or horizontally. Sometimes objects move at angles.

This object has a displacement of 8 m, but that's not Δx nor Δy.



So, let's call it ΔH, for the hypotenuse.

Of course, by knowing ΔH and θ, we can find Δx and Δy.



Δy = 8sin35º = 4.6 m

Δx = 8cos35º = 6.6 m

The kinematic equations work in one direction only: x, y, or H, as long as all of your quantities (v, a, x, etc) are in the same direction.

Horizontal Kinematic Equations	Vertical Kinematic Equations	Hypotenuse Kinematic Equations
$\Delta x = \frac{1}{2}(v_{xi} + v_{xf})t$	$\Delta y = \frac{1}{2}(v_{yi} + v_{yf})t$	$\Delta H = \frac{1}{2}(v_{Hi} + v_{Hf})t$
$v_{xf} = v_{xi} + a_x t$	$v_{yf} = v_{yi} + a_y t$	$v_{Hf} = v_{Hi} + a\Delta t$
$\Delta x = v_x t + \frac{1}{2}a_x t^2$	$\Delta y = v_{yi} t + \frac{1}{2}a_y t^2$	$\Delta H = v_{Hi} t + \frac{1}{2}a_H t^2$
$v_{xf}^2 = v_{xi}^2 + 2a_x \Delta y$	$v_{yf}^2 = v_{yi}^2 + 2a_y \Delta y$	$v_{Hf}^2 = v_{Hi}^2 + 2a_H \Delta y$
$\Delta x = v_{xf} t - \frac{1}{2}a_x \Delta t^2$	$\Delta y = v_{yf} t - \frac{1}{2}a_y t^2$	$\Delta H = v_{Hf} t - \frac{1}{2}a_H t^2$

Example 1 :

Part A: A rocket starts on the launch pad at rest at an angle of 43º. Its engines fire for 30 seconds giving 12 m/s² of acceleration. How far did the rocket go?

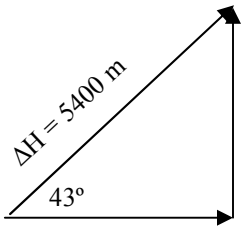
Part B: How much altitude did the rocket gain in that time?

Solution: Since the acceleration, velocity, time, and distance are all at 43º, then you are looking for the length of the *hypotenuse*. Just use one of your kinematic equations at 43º.

A: How far did the rocket go?

Variables:	Equation:
V_{Hi} = 0 m/s (at rest)	$\Delta H = V_{Hi}t + (1/2)a_H t^2$
a_H = 12 m/s²	$\Delta H = 0(30) + (1/2)12(30)^2$
t = 30 sec	$\Delta H = 0 + 6(900)$
ΔH = _____	ΔH = 5400 m

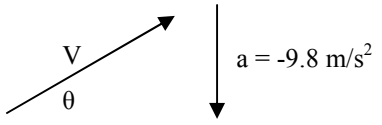
B: How much altitude did the rocket gain?



Δy = 5400sin43º = 3682.8 m

And finding Δx would be a simple matter.

IMPORTANT!



For projectiles you can't do this because V and a are in different directions.