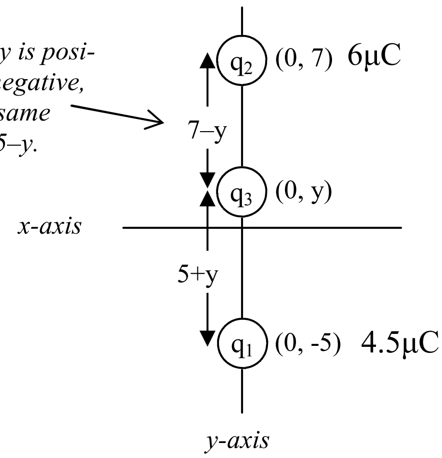
Example: Equilibrium Position Due to Two Charges

A 4.5μC charge is at y = -5m. A 6μC charge is at y = +7m. Where would a third charge be placed in between the two charges so that it experiences no acceleration (no net force)?

Notice that the assumption that y is positive is irrelevant. If y is really negative, the distance becomes 7+y. The same logic works for 5+y, becoming 5–y.



If it is not accelerating, $F_{net} = 0$. The coulomb force from each charge is equal in magnitude, but opposite in direction.

$$F_{net} = 0 = F_{q1} - F_{q2}$$

$$F_{q1} = F_{q2}$$

*Notice that since k_c and q_3 cancel **you don't need to know the charge**.*

$$k_c \frac{q_1 q_3}{r^2} = k_c \frac{q_2 q_3}{r^2}$$

$$\frac{q_1}{r^2} = \frac{q_2}{r^2}$$

The μC cancel, too.

$$\frac{4.5\mu C}{(5+y)^2} = \frac{6\mu C}{(7-y)^2}$$

$$\frac{4.5}{(5+y)^2} = \frac{6}{(7-y)^2}$$

Take the square root of both sides.

$$\frac{\sqrt{4.5}}{(5+y)} = \frac{\sqrt{6}}{(7-y)}$$

$$\frac{2.12}{(5+y)} = \frac{2.45}{(7-y)}$$

$$2.12(7-y) = 2.45(5+y)$$

$$14.84 - 2.12y = 12.25 + 2.45y$$

$$2.59 = 4.57y$$

$$y = .57$$

Important points:

1) You don't need to know the charge that is at equilibrium (no net charge). The sign of the charge is irrelevant because F_{net} can = 0 if there are two equal and opposite positive forces or negative forces. Also, the magnitude of the charge is irrelevant because the magnitude of each force will increase proportionally as the center charge increases.
2) It is the ratio of the two charges that matters, not the actual amounts. This is why the μCs don't matter. Actually, the above 4.5μC and 6μC could be 9μC and 12μC instead: their ratios are the same! (*See following proof.)*

$$\frac{\sqrt{9}}{(5+y)} = \frac{\sqrt{12}}{(7-y)}$$

$$\frac{3}{(5+y)} = \frac{3.46}{(7-y)}$$

$$3(7-y) = 3.46(5+y)$$

$$21 - 3y = 17.3 + 3.46y$$

$$3.7 = 6.46y$$

$$y = .57$$

Same position!